

# How computers have changed how we do Physics: Chaos and Climate

MYLES ALLEN

Environmental Change Institute, School of Geography and  
the Environment &

Atmospheric, Oceanic & Planetary Physics

University of Oxford

[myles.allen@ouce.ox.ac.uk](mailto:myles.allen@ouce.ox.ac.uk)

# So what have computers ever done for us?



- Chaos in weather and seasonal forecasting
- Understanding ocean climate
- Weather and climate change
- Centennial climate prediction

# Edward Lorenz, 1917-2008

JOURNAL OF THE ATMOSPHERIC SCIENCES

VOLUME :

## Deterministic Nonperiodic Flow<sup>1</sup>

EDWARD N. LORENZ

*Massachusetts Institute of Technology*

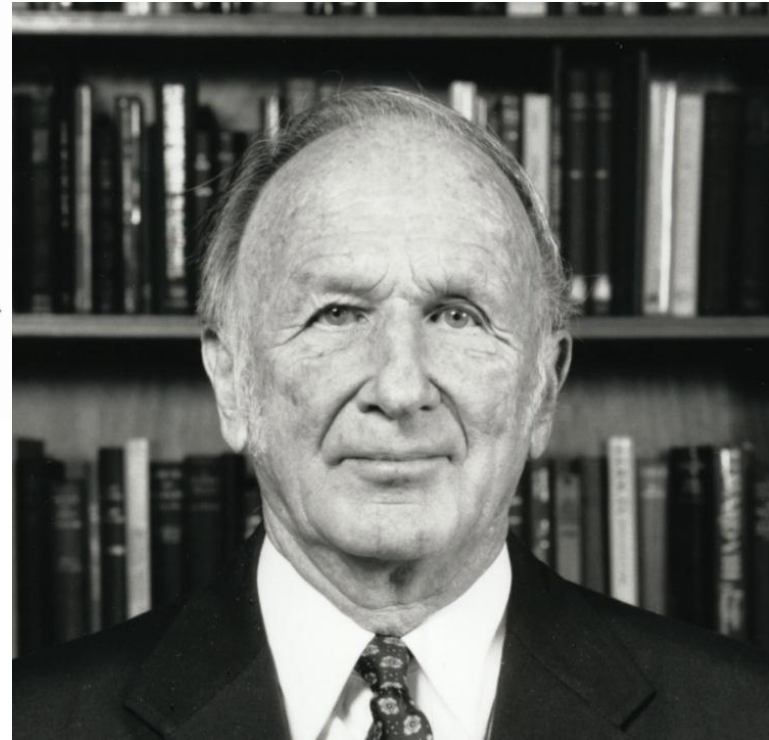
(Manuscript received 18 November 1962, in revised form 7 January 1963)

### ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions. A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

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The feasibility of very-long-range weather prediction is examined in the light of these results.



# The Lorenz (1963) system

Three dimensional, deterministic:

$$\mathbf{x} = \mathbf{x}(x, y, z)$$

with the Jacobian

$$\dot{\mathbf{f}}(\mathbf{x}) = \mathbf{J}(\mathbf{x})$$

$$\mathbf{J} = \begin{pmatrix} -\sigma & \sigma & 0 \\ r - z & -1 & -x \\ y & x & -b \end{pmatrix}$$

# Volume contraction in the L'63 system

Phase space volume: occupied by a set of points in L'63

$$\begin{aligned}\dot{V} &= \int_S \mathbf{f} \cdot \mathbf{n} \, dA \\ &= \int_V \nabla \cdot \mathbf{f} \, dV \\ &= \int_V \text{tr}(\mathbf{J}) \, dV \\ &= (-\sigma - 1 - b)V\end{aligned}$$

So it's dissipative:

$$V(t) = V(0)e^{(-\sigma-1-b)t}$$

# Stability and error growth in the L'63 system

For  $r > r_H = \frac{S(S+b+3)}{S-b-3}$  we have

No stable fixed points & no stable periodic orbits.

So volumes contract, but not to a point...

# Stability and error growth in the L'63 system

What happens to a small perturbation,  $d\mathbf{x}$ ?

$$\begin{aligned}d\mathbf{x}_{t+dt} &= (\mathbf{I} + \mathbf{J} dt) d\mathbf{x}_t \\ &= \mathbf{U}(\mathbf{I} + \mathbf{W} dt) \mathbf{V}^T d\mathbf{x}_t \text{ where}\end{aligned}$$

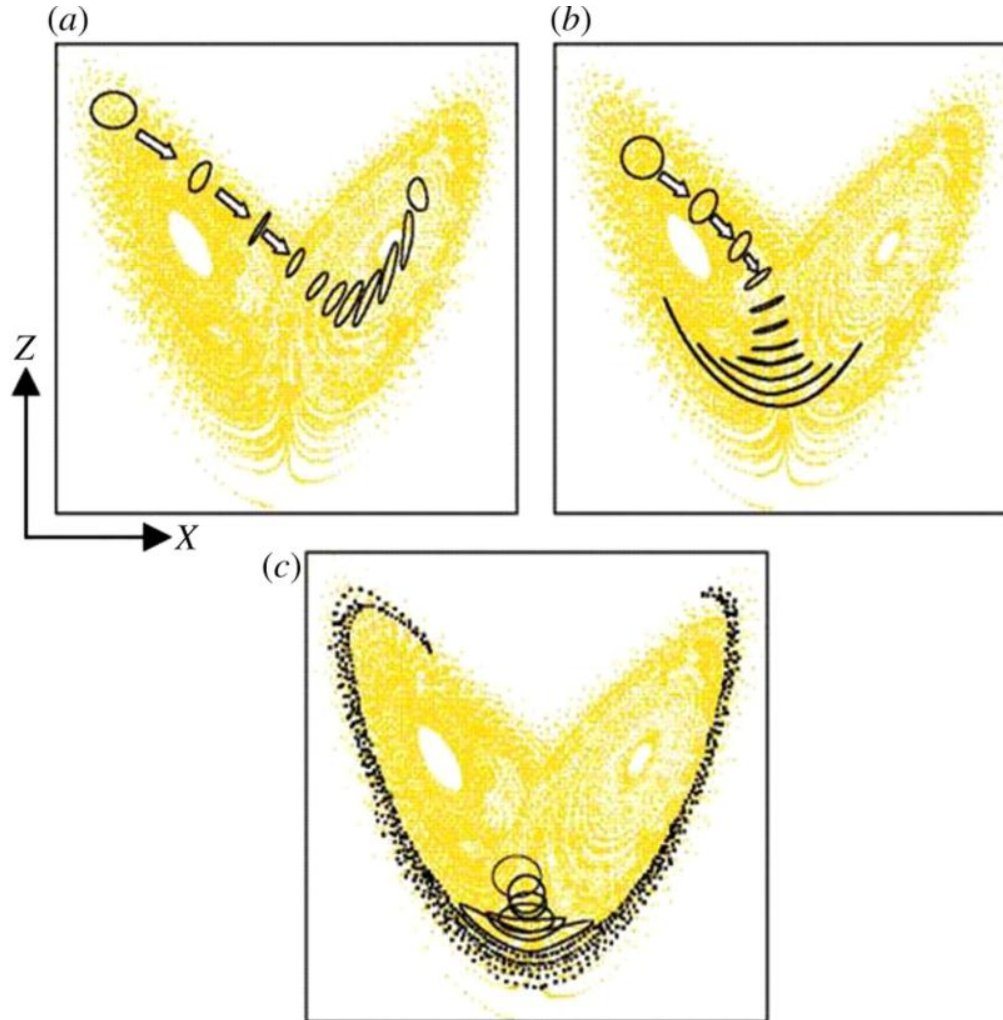
$\mathbf{J} = \mathbf{U}\mathbf{W}\mathbf{V}^T$  is the singular value decomposition.

So: a perturbation aligned with any basis vector  $\mathbf{v}_j$

grows or shrinks by a factor  $(1 + w_j dt)$

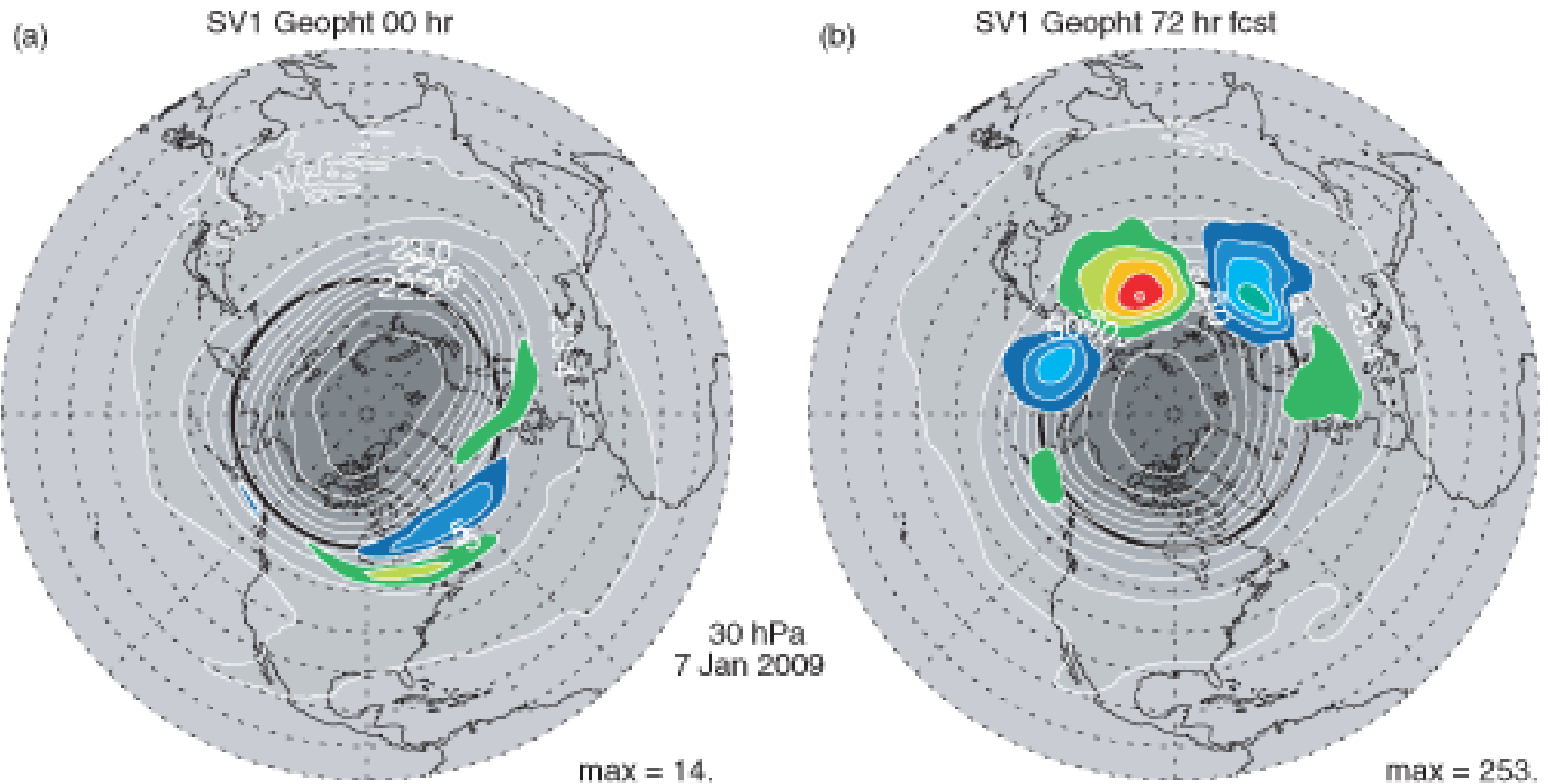
and rotates to align with  $\mathbf{u}_j$

# Stability and error growth in the L'63 system





# Non-linear evolution of singular vectors during the January 2009 stratospheric sudden warming



**L. Coy and C. Reynolds (2013)**

Quarterly Journal of the Royal Meteorological Society

Volume 140, Issue 680, pages 1013-1024, 20 JUN 2013 DOI: 10.1002/qj.2181

<http://onlinelibrary.wiley.com/doi/10.1002/qj.2181/full#fig3>

# Scales of motion in the ocean

(courtesy of NASA and ECCO2 ocean reanalysis project)

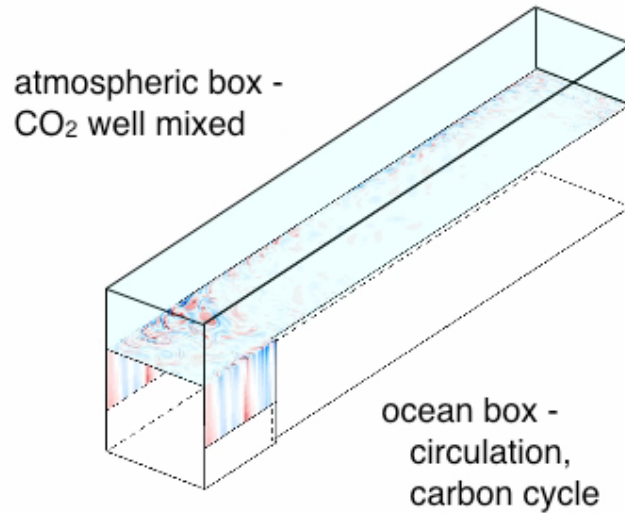
- <http://svs.gsfc.nasa.gov/vis/a010000/a010800/a010841/index.html>

# Ocean eddies and ocean climate

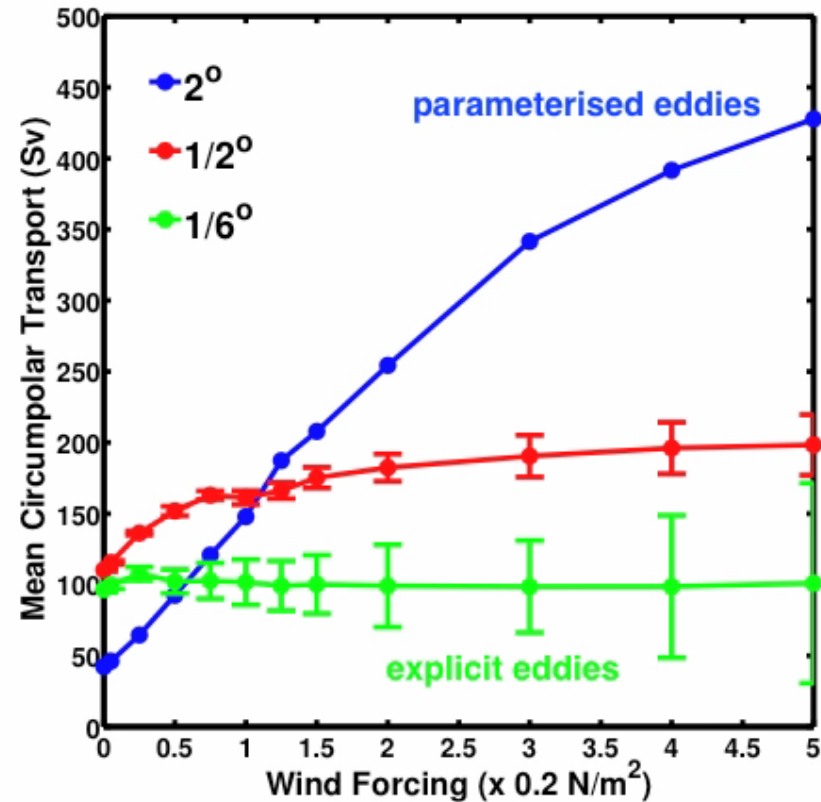
(courtesy of David Marshall, Atmospheric, Oceanic & Planetary Physics)

Ocean models behave very differently with **parameterised** and **explicit** eddies

(Munday et al., 2013)



“eddy saturation”



# So what about the weather a little closer to home?



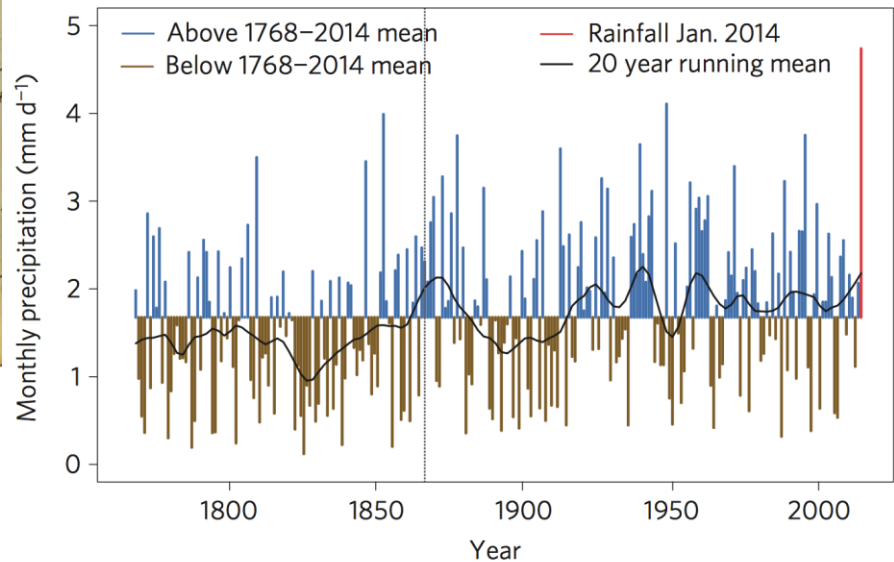
Storms and floods in January 2014  
N. Schaller et al, *Nature Climate Change*,  
February 2016



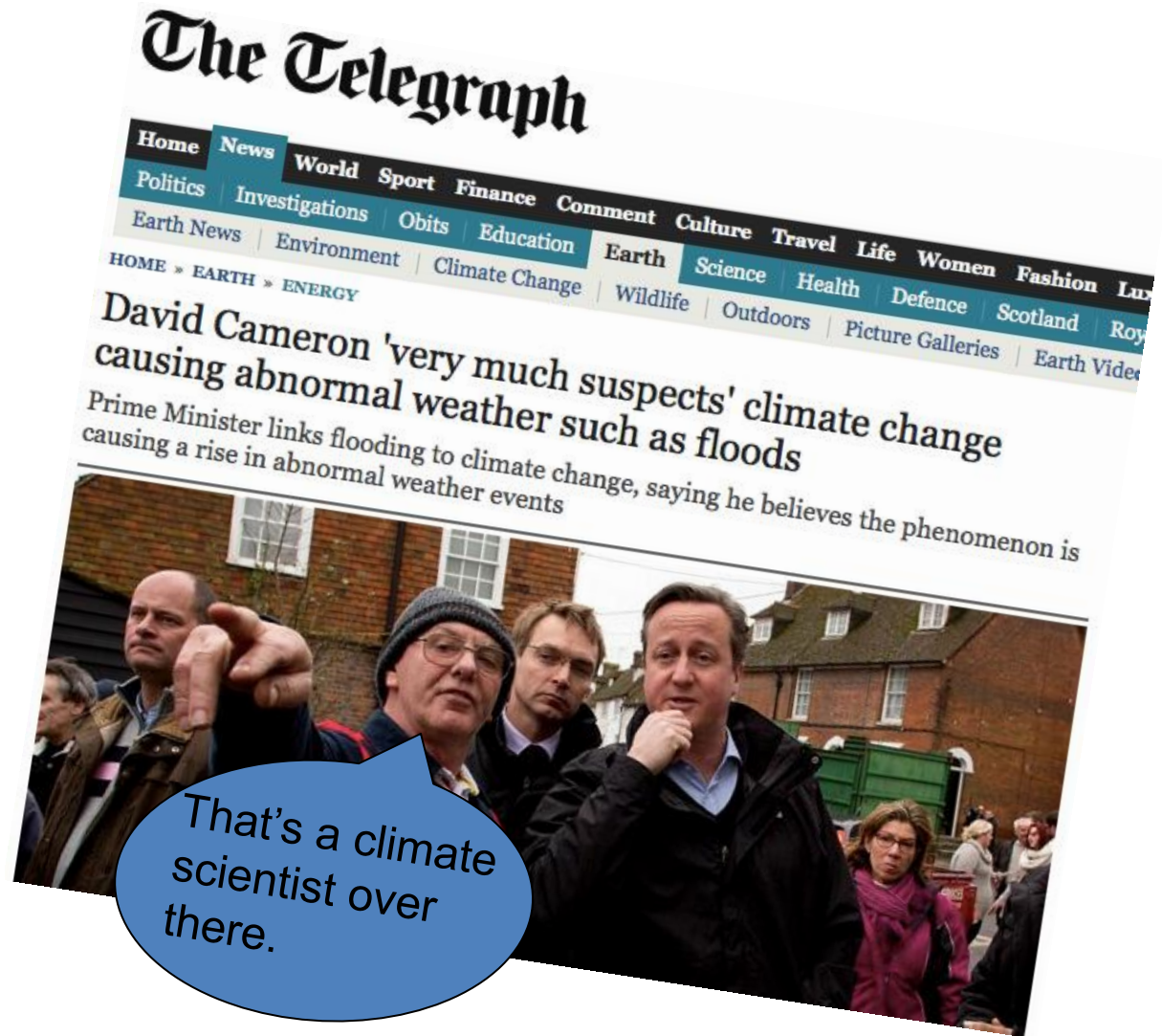
# The wettest January and the wettest winter in the world's longest daily weather record: Oxford, 1767-present



Ian Ashpole making history

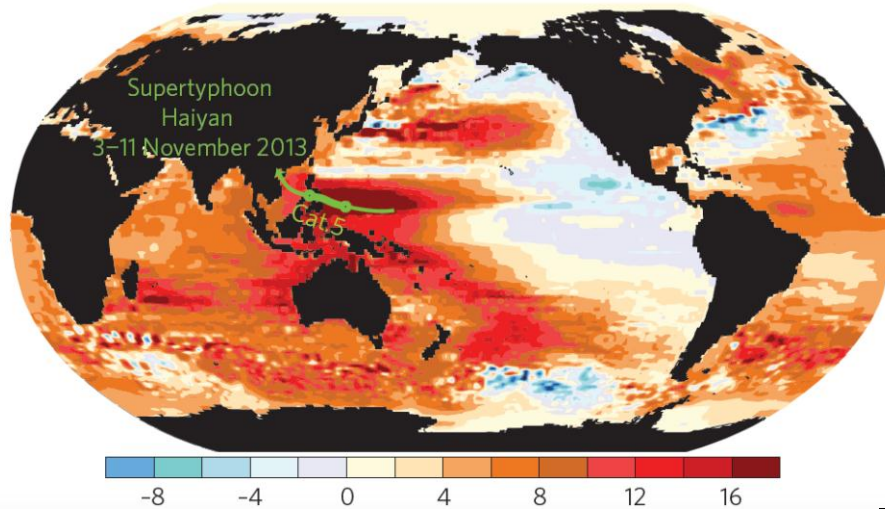


# With the usual impacts



# Is climate change “making storms stronger”?

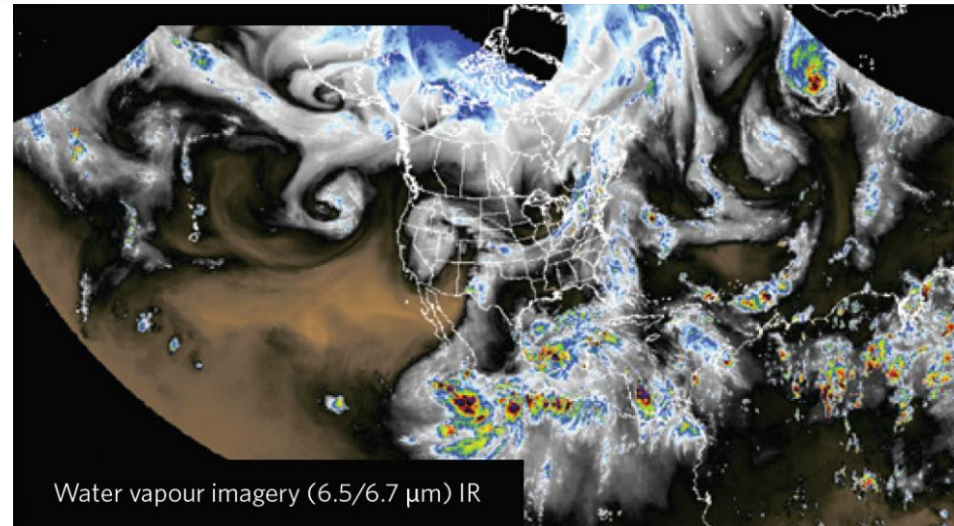
Global mean sea-level trend August 1993–July 2013



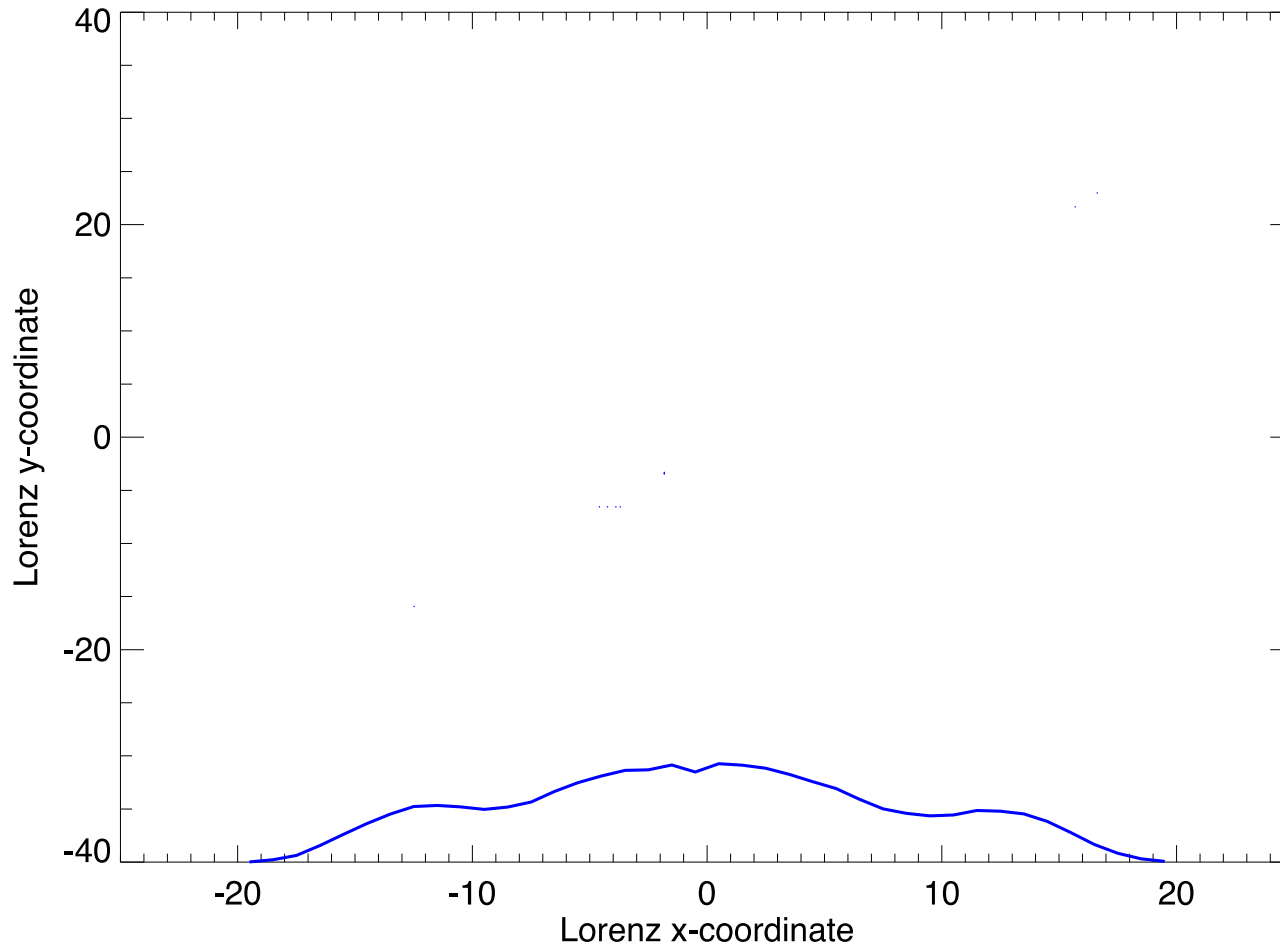
Sea level rise and Typhoon Haiyan

Examples from Trenberth *et al*,  
*Nature Climate Change*, 2015

Atmospheric moisture and 2013 Boulder floods

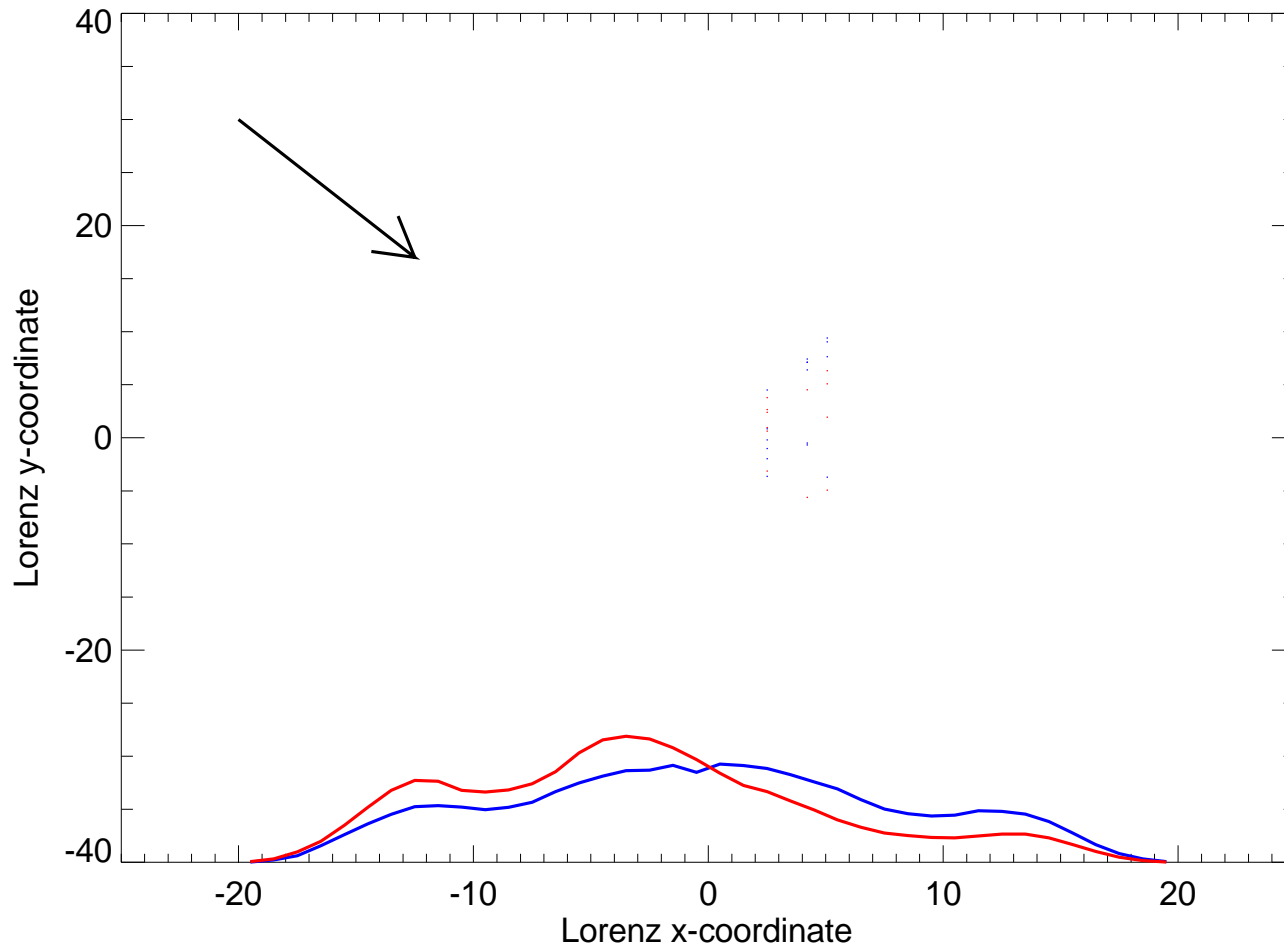


# Back to L'63, as modified by Tim Palmer (2003)

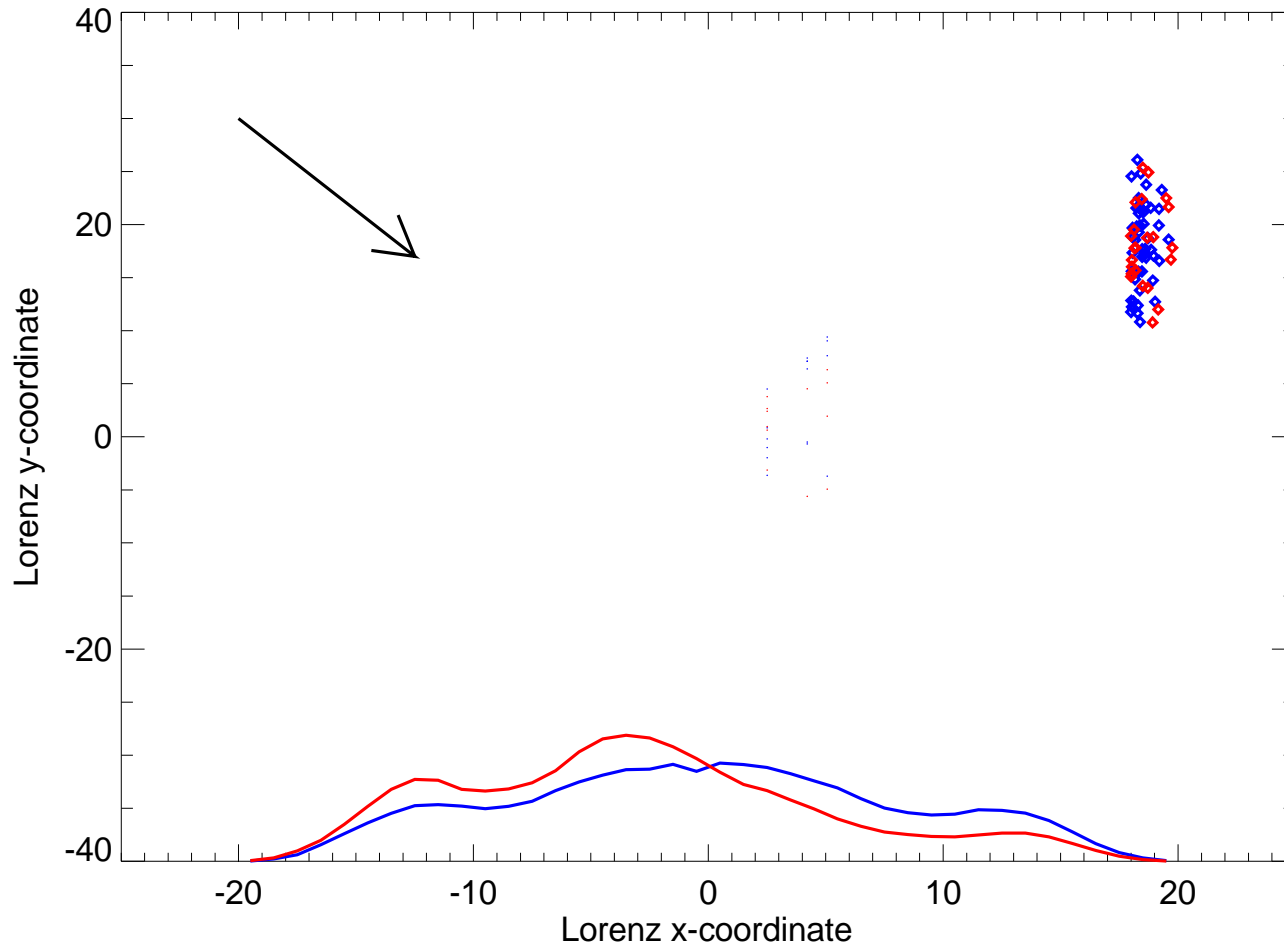




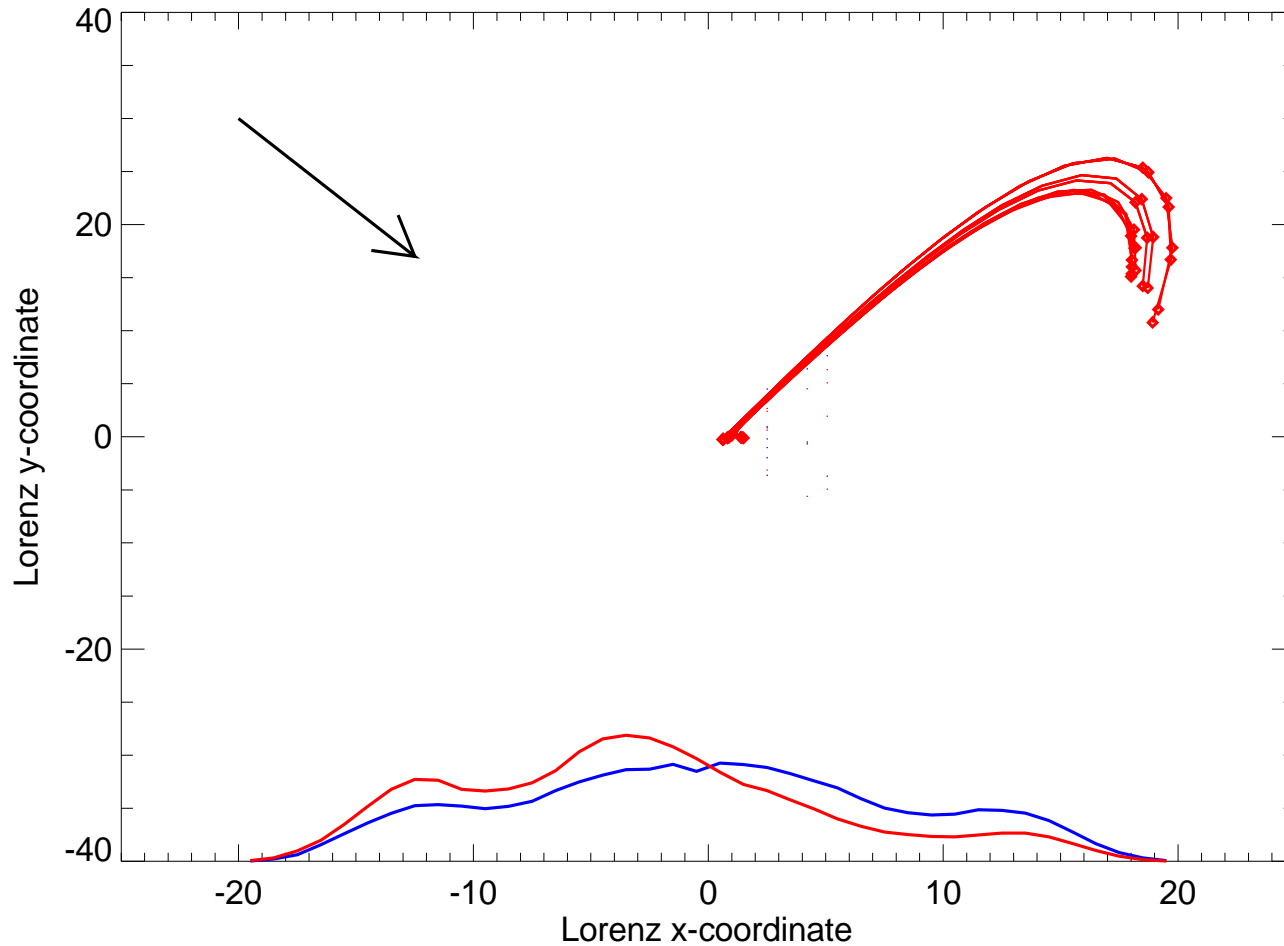
# Impose an external forcing



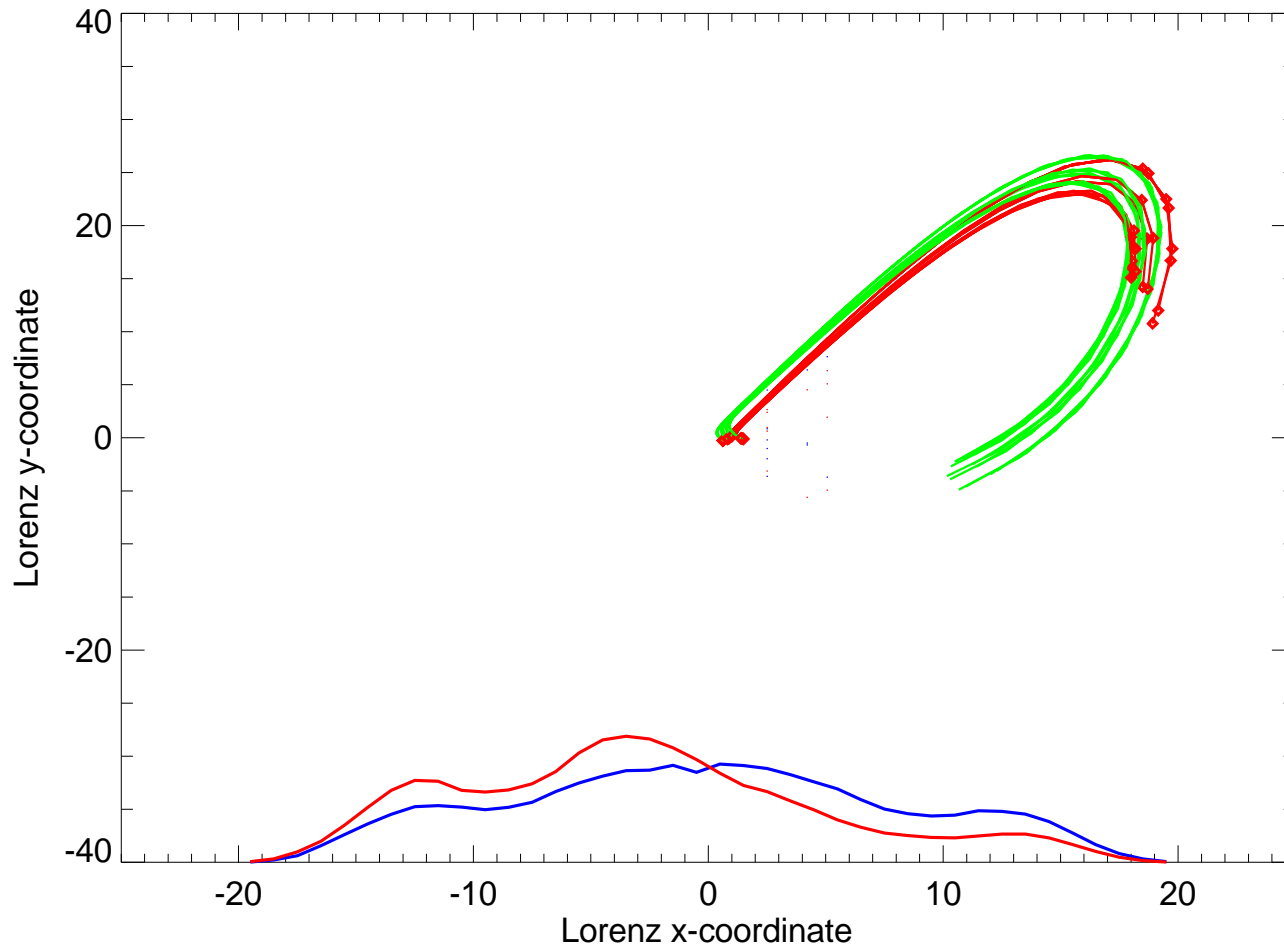
# Forcing *halves* the risk of x-coordinate exceeding a threshold



# Now consider the role of external forcing in each of these extreme events



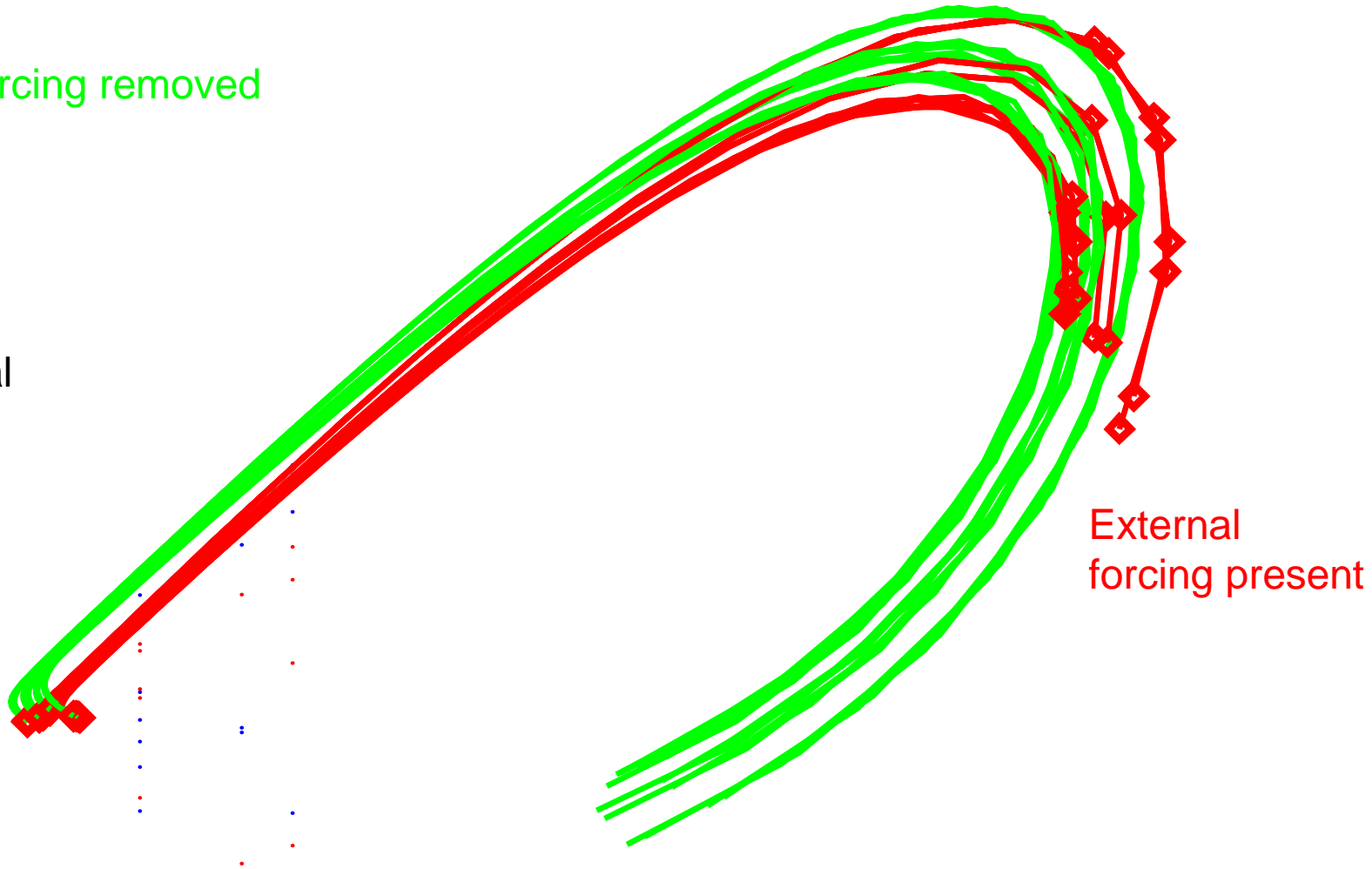
# External forcing *increases* magnitude of individual events



# External forcing *increases* magnitude of individual events

External forcing removed

Same initial conditions



So even if climate change makes individual storms stronger, it isn't necessarily loading the weather dice towards stronger storms

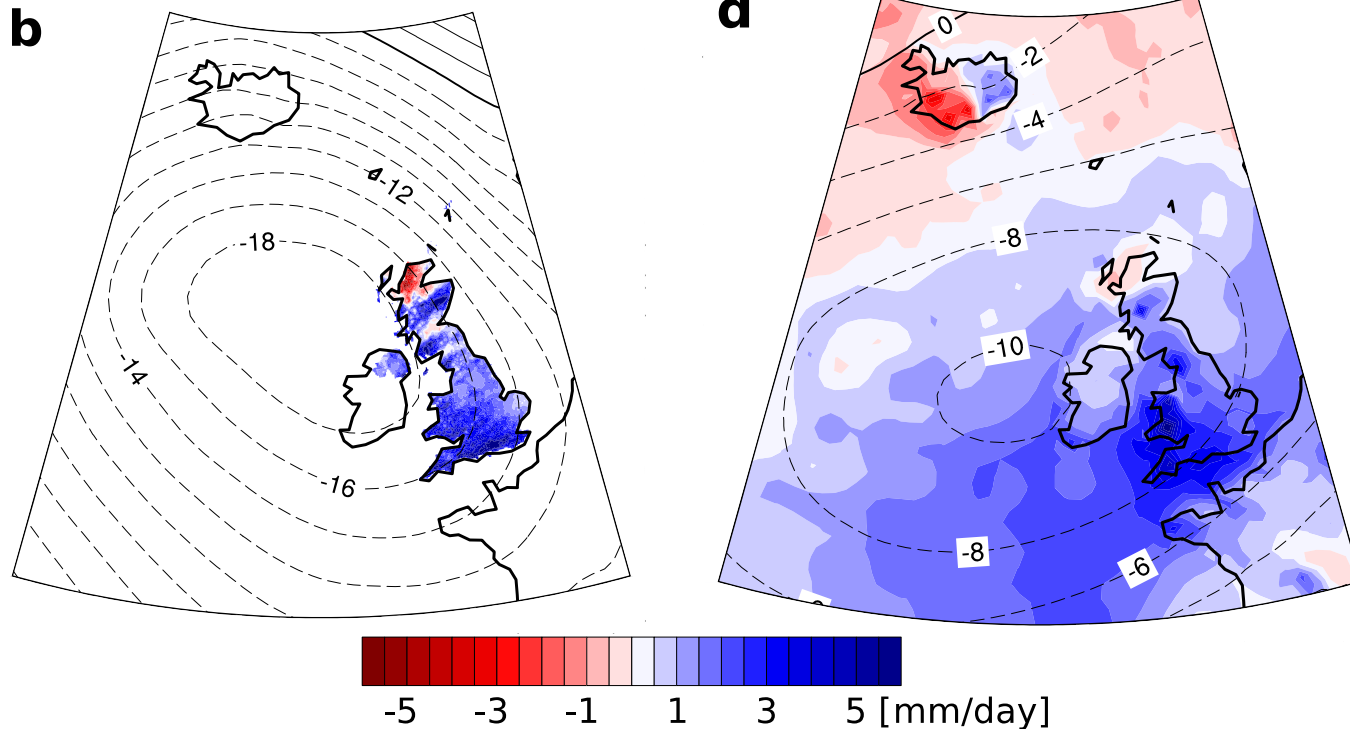


Allen, M. R., Liability for Climate Change, *Nature*, 2003

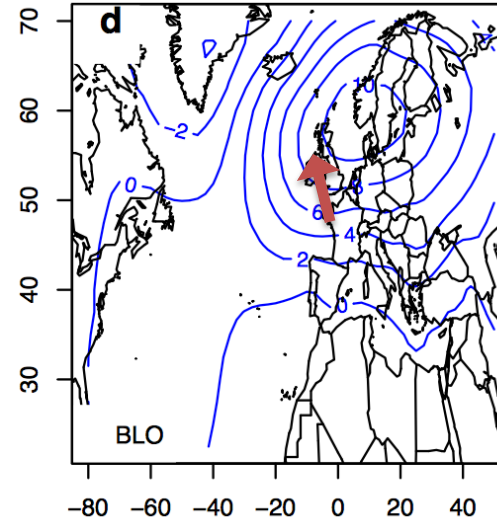
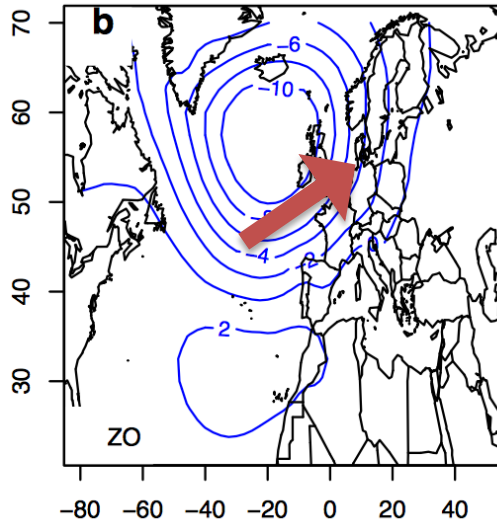
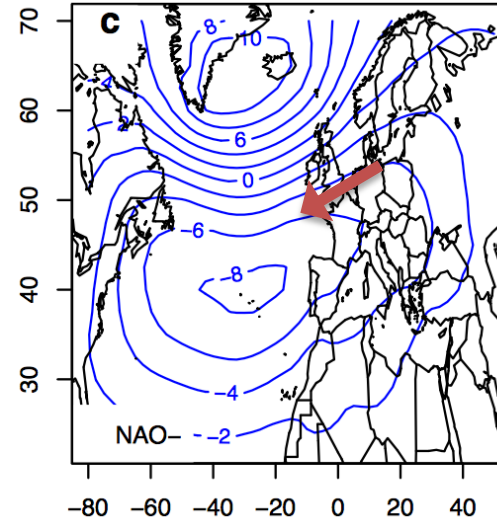
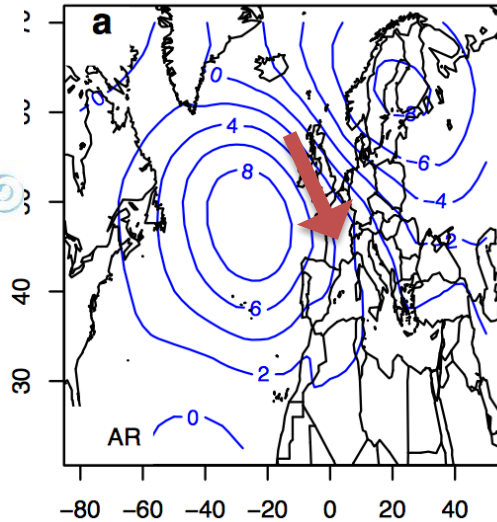
# *In silico* experiments simulating extreme weather

January 2014  
observed precipitation  
& MSLP anomalies

Precipitation and MSLP  
anomalies in the wettest 1% of  
the “actual conditions” ensemble

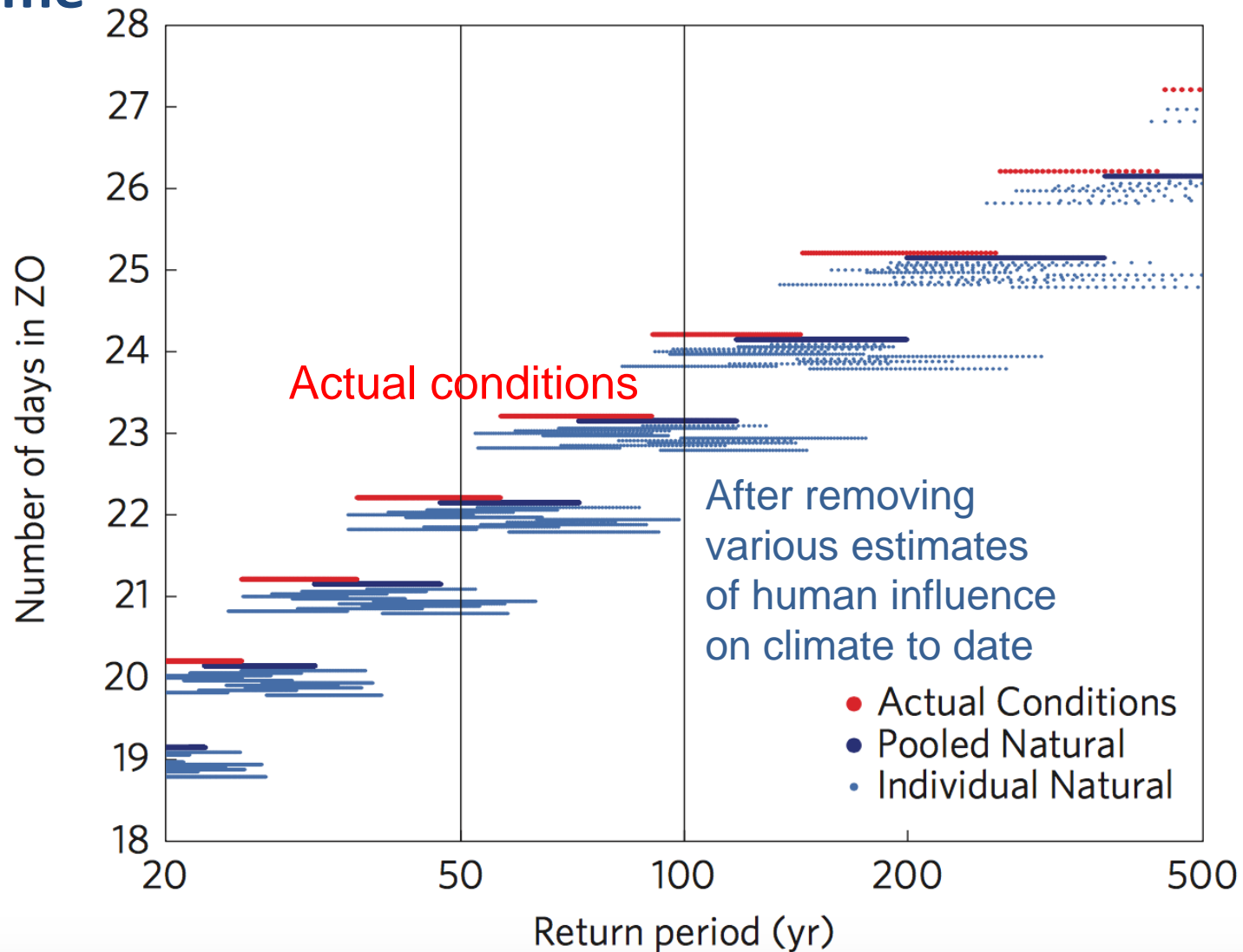


# Weather regimes in the North Atlantic in winter

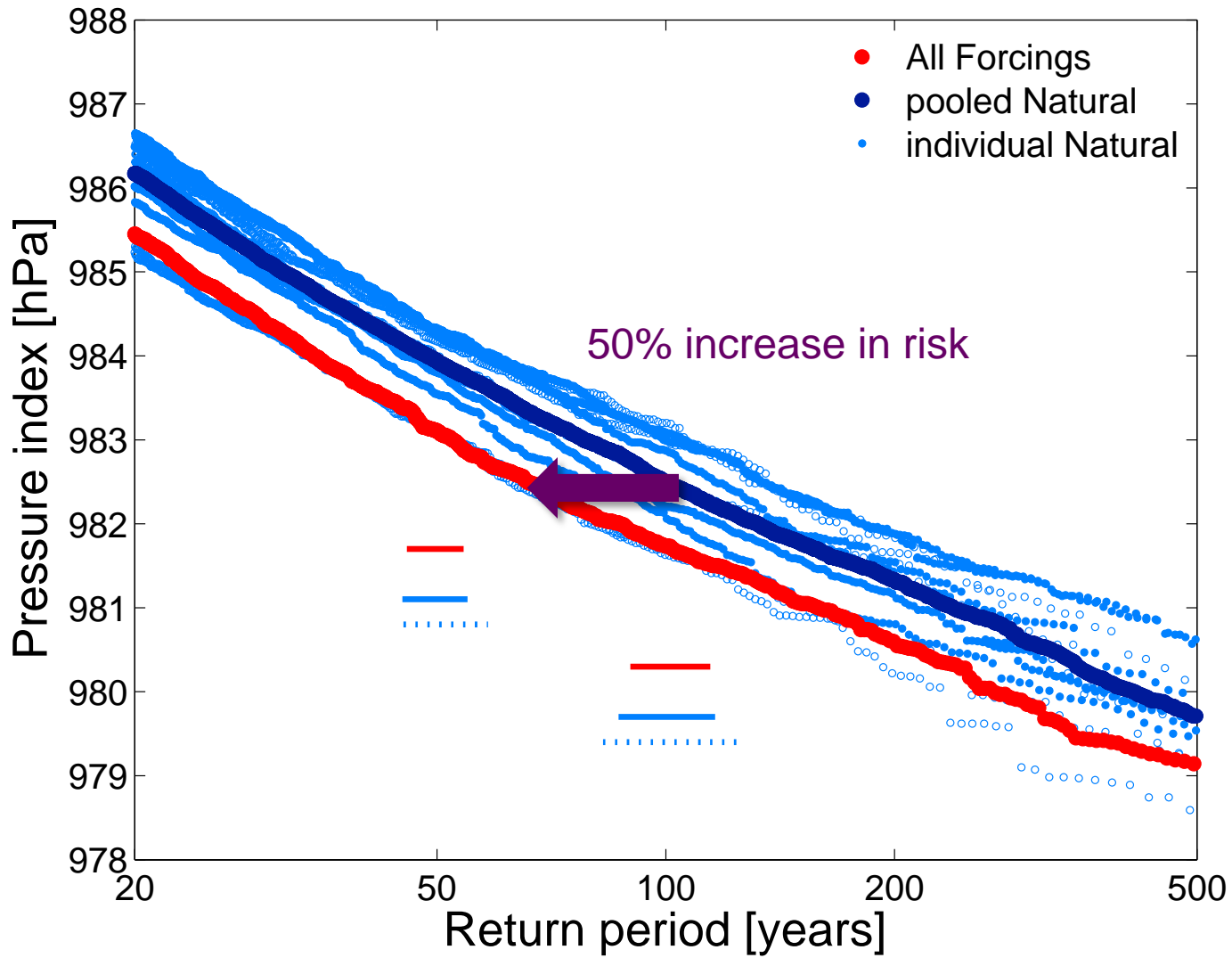




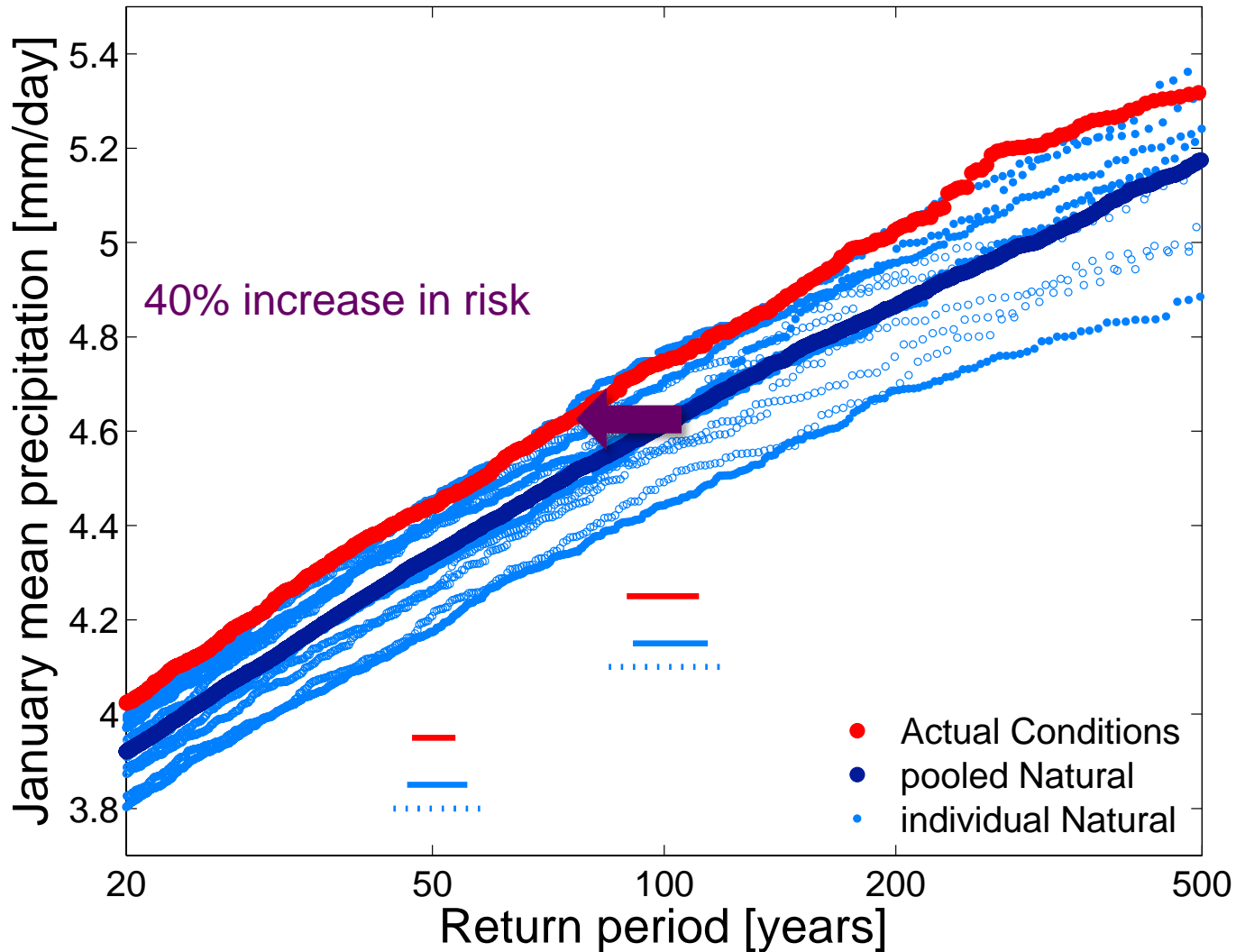
# Small increase in number of days in the “zonal” regime



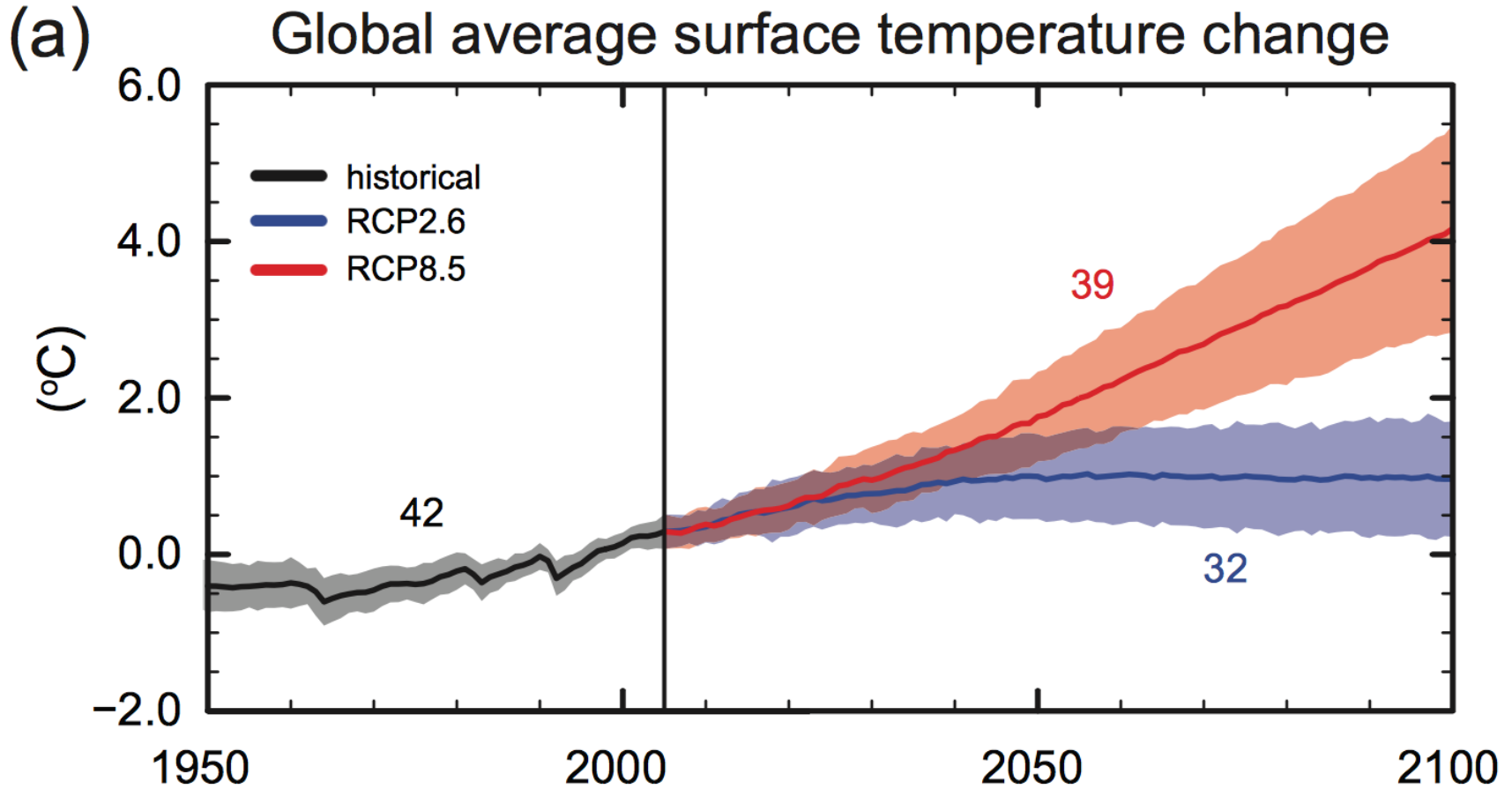
# Change in risk of low pressure at 60N,20W



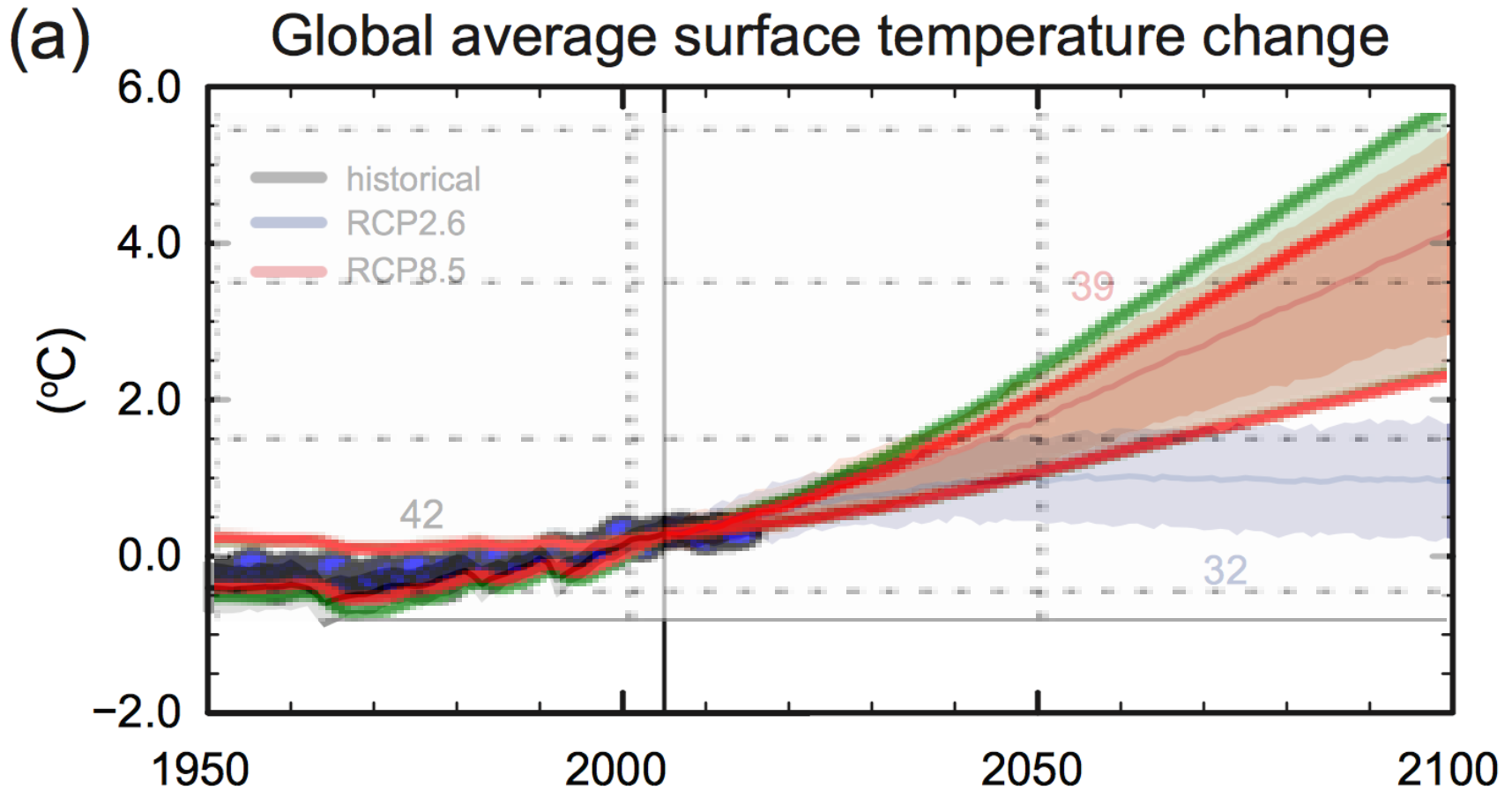
# Change in risk of high Southern England rainfall



# So what about the flagship climate prediction problem?

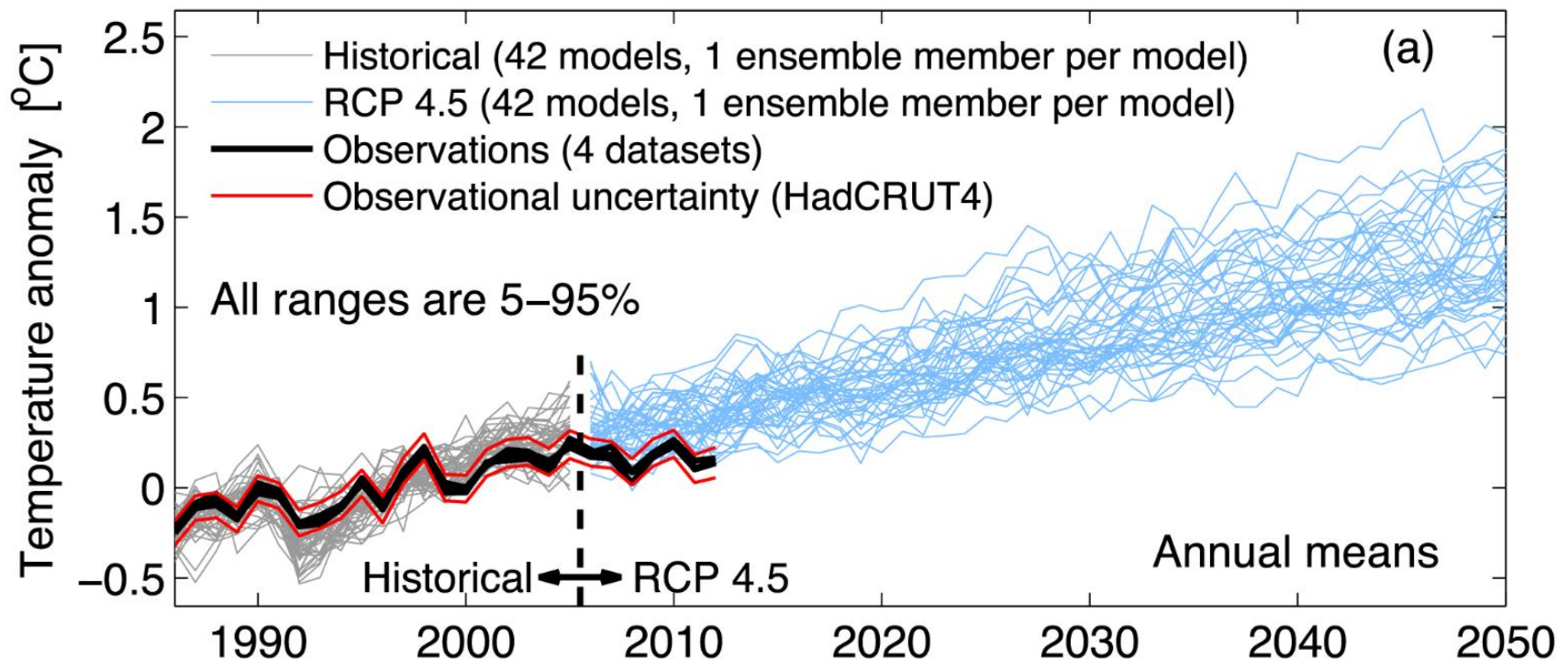


# So what about the flagship climate prediction problem? Compare to a 1980s 2-parameter ODE

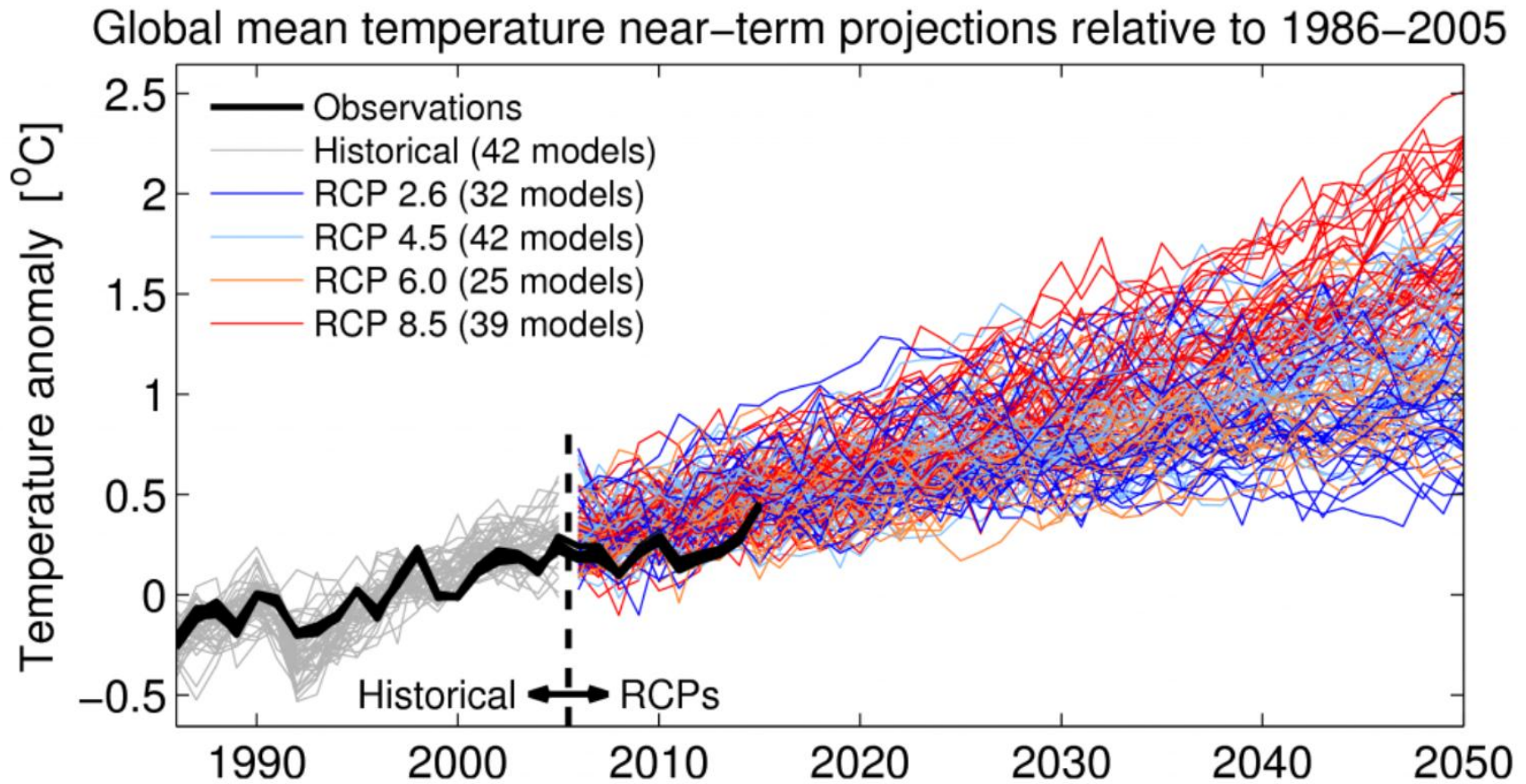


# And frankly, these models can give more grief than they are worth...

Global mean temperature projections (RCP 4.5), relative to 1986–2005

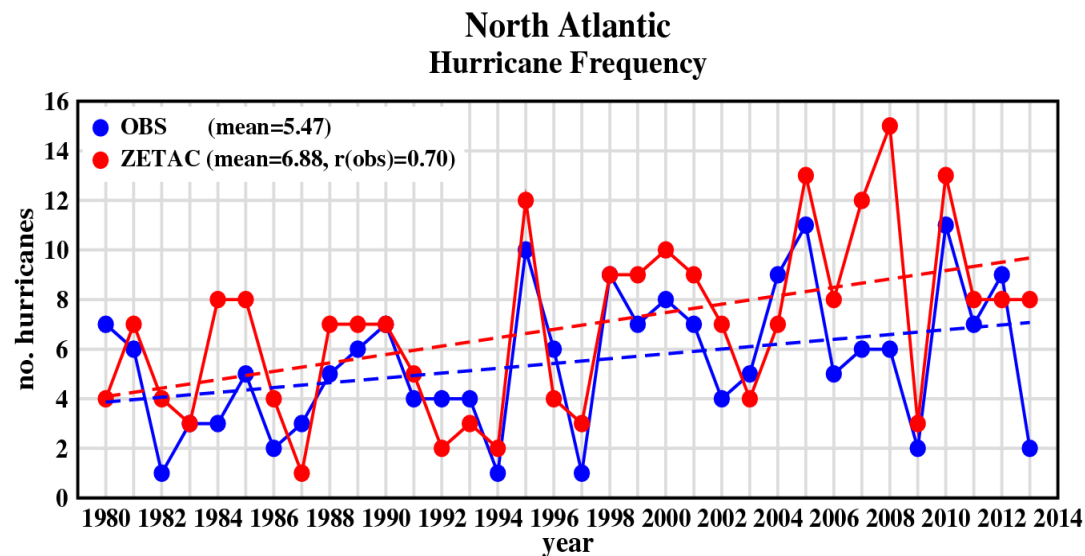


# Although the update looks a little less exciting (courtesy of Ed Hawkins)



# Computers have clearly transformed weather & regional climate prediction

- See this link for an impressive demonstration of how high-resolution computer modelling has transformed our understanding of year-to-year variations in Atlantic hurricane risk: [http://www.gfdl.noaa.gov/bibliography/related\\_files/Knutson0701.pdf](http://www.gfdl.noaa.gov/bibliography/related_files/Knutson0701.pdf)
- And just to prove the didn't get lucky in 2007, here is an updated comparison of artyes of  
Tom Knutson:





But (fortunately) we don't actually seem to need them to predict global temperature...

